Master 2: *International Centre for Fundamental Physics*

INTERNSHIP PROPOSAL

Laboratory name: **Laboratory of Condensed Matter Physics**
CNRS identification code: **UMR7643**
Internship director'surname: **Denis GREBENKOV**
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Web page: **http://pmc.polytechnique.fr/pagesperso/dg**
Internship location: **Laboratory of Condensed Matter Physics, Ecole Polytechnique, Palaiseau, France**
Thesis possibility after internship: YES
Funding: NO (but it is possible to apply for a scholarship from Ecole Polytechnique)

Conformal mapping and first passage time statistics

How long does it take for an animal to find food or for a protein to find the specific gene sequence on a DNA chain? This fundamental question arises in many physical, chemical and biological systems[1]. In mathematical terms, one aims at characterizing the random moment (called the first passage time, FPT) at which a given stochastic process reaches a particular state for the first time. The problem has attracted a considerable attention of mathematical and theoretical physics communities during the past decade, particularly because of its important biological implications. Significant progress has been recently made in the analysis of the mean FPT for a diffusive process in a planar domain by using conformal mapping[2]. In fact, mapping a given simply connected planar domain onto the unit disk yields a universal formula for the mean FPT. This formula reveals deep connections between the mean FPT and the harmonic measure, which characterizes the accessibility of a boundary of the domain by Brownian motion[3] and plays the role of the natural small parameter in the mean FPT problem.

The internship focuses on further theoretical and numerical studies of this fascinating problem. First, one needs to formalize the relation between the harmonic measure and the mean FPT. Second, the derived formula offers an efficient numerical way to compute the mean FPT via conformal mapping. A practical implementation of this approach will open promising computational facilities. Third, the formula can potentially be generalized to domains with several exit regions on the boundary or to domain with holes. Fourth, the conformal mapping approach revealed some conceptual limitations in former asymptotic results for planar domains. Although similar limitations are expected in higher dimensions, there is currently no clue how to overcome them. In this situation, numerical simulations in complex three-dimensional domains present a natural first step towards a better understanding of these first passage phenomena.

The candidate is expected to have a solid background in theoretical/mathematical physics or/and mathematics and be skilled in numerical simulations (e.g., in Matlab). The internship can potentially be continued as a PhD thesis under the condition of obtaining a PhD scholarship (in order to be able to apply for such a scholarship at Ecole Polytechnique, the candidate should have excellent notes).

Bibliography

[1] Y. Lanoiselée, N. Moutal, and D. S. Grebenkov, *Diffusion-limited reactions in dynamic heterogeneous media* Nature Comm. **9**, 4398 (2018) [online: https://pmc.polytechnique.fr/pagesperso/dg/publi/2018_10.pdf]
[2] D. S. Grebenkov, *Universal formula for the mean first passage time in planar domains*, Phys. Rev. Lett. **117**, 260201 (2016) [online: https://pmc.polytechnique.fr/pagesperso/dg/publi/2016_07.pdf]
[3] D. S. Grebenkov, *What Makes a Boundary Less Accessible*, Phys. Rev. Lett. **95**, 200602 (2005) [online: https://pmc.polytechnique.fr/pagesperso/dg/publi/2005_01.pdf]

Please, indicate which speciality(ies) seem(s) to be more adapted to the subject:

Condensed Matter Physics: YES Macroscopic Physics and complexity: YES
Quantum Physics: YES Theoretical Physics: YES