Internship proposal: Statistical physics of Physics Informed Neural Networks

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Non-linear partial differential equations (PDEs) arise everywhere in science and engineering. Despite their ubiquity, exact solutions are known only in a few special cases; in the vast majority of situations, PDEs must be solved numerically. This is generally challenging because non-linearities can produce boundary layers or blow-up — think of the Navier–Stokes equations and the corresponding Millennium Prize Problem — which in turn trigger numerical instabilities. For this reason, a well-developed branch of applied mathematics is devoted to designing numerical techniques that yield stable results and are now used routinely across disciplines.

In recent years, the rise of neural networks has radically transformed the landscape of numerical methods. Since neural networks are universal approximators, researchers have sought to exploit their expressive power to parametrize solutions of non-linear PDEs. These approaches are now revolutionizing the field and are being used to uncover scaling regimes and non-linear phenomena in PDEs. Yet, as in other areas where neural networks are having a disruptive impact, we still understand very little about how the performance of these methods and their training strategies depends on the structure of the problem, the regularity of the target solution, and the loss function used during training.

The objective of this internship is to study simple models of physics-informed neural networks that can be analyzed using techniques from statistical physics. We will employ tools such as dynamical mean-field theory to address training dynamics and connect the problem to large deviation theory.